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## Simulation of Orthogonal Cutting with Smooth Particle Hydrodynamics

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# **Simulation of Orthogonal Cutting with Smooth Particle Hydrodynamics**

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## **ABSTRACT**

There is an active literature on the simulation of cutting processes through finite element methods. Such efforts are motivated by the enormous economic importance of machining processes and the desire to adjust processes so as to optimize product and throughput, but suffer from some difficulties inherent to the finite element method. An alternative approach, which appears to overcome most of those difficulties, is that of Smooth Particle Hydrodynamics (SPH). Though some finite element work is reviewed here, the focus of this paper is on the demonstration of the SPH technique of to simulate orthogonal cutting.



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# Simulation of Orthogonal Cutting with Smooth Particle Hydrodynamics<sup>\*</sup>

## Introduction

Classic methods such as slip lines, e.g. Merchant [1] and Lee and Shaffer [2], have been used in the past to explore the cutting process and to predict cutting forces. These methods require severe kinematic assumptions and are somewhat limited to simplified material (constitutive) response. Resorting to weighted residual techniques involving basis functions with only local support is therefore desirable and often necessary to solve the field equations. Several efforts have been made using the finite element method, see e.g. Strenkowski [3, 4], Shih [5]. However, several deficiencies of the finite element method in the simulation of cutting remain:

- the propensity of individual elements to be turned inside out as the material undergoes very large deformations in the vicinity of the cutting tool;
- the difficulty of accommodating material failure (separation) within the finite element mesh;
- the uncertainty and ambiguity of implementing material failure criteria during cutting.

Recently, Maraush [6] has shown that aggressive re-meshing may be used to circumvent severe mesh distortion. However, there is a substantial cost associated with re-meshing and there is uncertainty in preserving material state variables in the re-mapping process. Approaches to addressing the second of these issues are discussed in the literature and a good review can be found in [6], yet in general this issue awaits satisfactory resolution.

Because of the above fundamental uncertainties, most simulation focuses on orthogonal cutting, where these research issues can be attacked with relatively little expense.

In this paper, the natural manner in which the SPH method meets many of these challenges is presented. The issue of material failure criteria during cutting appears to be as difficult in this approach as in the others.

## Smooth Particle Hydrodynamics

The method of “smooth particle hydrodynamics” is reviewed in Benz [7] and discussed in other perspectives in that paper. Just a minimal discussion of this technique is given here for the purpose of clarity and continuity. SPH consists of a collection of nodes each having

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physical degrees of freedom. The nodes have no fixed connectivity. Instead, the instantaneous connectivity is determined from proximity of nodes to near neighbors, using a search technique. The nodal contribution to the field quantity,  $f$ , at a location  $\hat{x}$  is:

$$f(\hat{x}) = f_n W(\hat{x} - \hat{x}_n, h) \quad (1)$$

where  $f_n$  is the field value associated with node  $n$  and  $W(\hat{x} - \hat{x}_n, h)$  is the corresponding shape function. The shape function is centered on the node  $n$  and can take many forms (several are discussed in [7]) but must satisfy specific requirements. It must be spherically symmetric and integrate to unity, i.e.

$$\int W\left(\frac{\hat{r} - \hat{x}_n}{h}\right) d\vec{r} = 1, \quad (2)$$

decay monotonically from its reference node  $\hat{x}_n$ , and be zero beyond a distance of  $h$ , i.e. have local support. The shape function used here is a cubic spline:

$$W(\hat{x} - \hat{x}_n, h) = \frac{h}{\pi|\hat{x} - \hat{x}_n|} \begin{cases} 1 - \frac{3|\hat{x} - \hat{x}_n|^2}{2h^2} + \frac{3|\hat{x} - \hat{x}_n|^3}{4h^3} & \text{if } 0 \leq |\hat{x} - \hat{x}_n| \leq h \\ \frac{1}{4}(2 - |\hat{x} - \hat{x}_n|)^3 & \text{if } h \leq |\hat{x} - \hat{x}_n| \leq 2h \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

One of the most important features of SPH is the manner in which the gradient of the field quantity is computed. It can be shown that the contribution of a nodal value to the gradient of the field at  $\hat{x}$  is  $f_n \nabla W(\hat{x} - \hat{x}_n, h)$ . Note that the gradient remains meaningful no matter how neighboring nodes are rearranged. In this way the SPH method obviates the problem of element distortion common to finite elements in problems of large shear. Finally, point masses are associated with each node (also referred to as particles).

In the case of the explicit transient dynamic simulation such as used here, the momentum equation:

$$0 = \rho \frac{\partial}{\partial t}(v) + \nabla \cdot \sigma \quad (4)$$

is solved using a central difference time integrator. In equation (4),  $\rho$  is the material density,  $v$  is the material velocity and  $\sigma$  is the Cauchy stress. Using the above gradient methods for evaluating the strain rate from the velocity field, and using appropriate constitutive equations, stress increments can be calculated. Finally, the divergence of the stress field is evaluated, and each node occupying a volume  $V$  with mass  $\rho V$  is accelerated accordingly. In this manner the motion of the nodes is driven by the mechanics being simulated. We

note that relevant field quantities are re-evaluated at each time step and at each 1/2 time step as required by the central difference time integrator.

Unfortunately, a recent stability analysis of SPH has shown the SPH gradient operator to be only conditionally stable. A numerical technique called conservative smoothing seems to stabilize the SPH algorithm in explicit transient dynamics applications at the expense of introducing some numerical diffusion. Further details on this can be found in Swegle, et. al. [8, 9] and Wen et. al. [10].

## Material Failure Modeling

Calculating separation through material failure modeling is another of the difficult issues in cutting simulation. One method used in finite element analysis to address this issue is to evaluate damage on each element and to remove those that suffer damage beyond some specified level. The obvious deficiency of this approach is that a significant part of the material volume is removed from the calculation unless an excessively fine mesh is used.

Another method used to attempt to capture material failure requires postulating the locus of material separation. A special layer of contact elements are placed on that path and separation is permitted as some break-away load is achieved. The deficiencies of this approach are not only that it lacks theoretical rigor, but that implementation of it in the context of large-deformation plasticity is a logistical ordeal.

The kinematics of material separation are accommodated in SPH in a manner that neither involves the loss of material, requires foreknowledge of the locus of separation, nor requires special numerical treatment. Material damage is incorporated at SPH nodes through a loss of cohesion as neighboring SPH particles separate from each other. This comes about because once those particles are more than the critical distance,  $h$ , from each other, each particle no longer contributes to the strain calculated at the other and the corresponding cohesive component of the stress disappears.

Though the SPH method offers advantages over the finite element methods in terms of accommodating the large deformations and the kinematic issues of material failure modeling, the problem of defining physical criteria (such as failure strain, failure stress, or failure energy) for material separation remains. That problem is a continuing topic for research.

## Cutting Simulations

Orthogonal cutting problems involve passing a relatively hard tool through a softer work-piece, and this process is appropriately simulated by solving the nonlinear governing equations in a region very near the cutting tool. (See Figure 1). In this local problem, kinematic boundary conditions are applied suitably far from the cutting tool/work-piece interface, providing relative motion between work-piece and tool. As a result, feed force,

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cutting force and cutting tool moment reactions are evaluated at boundaries that are also far from the cutting interface.

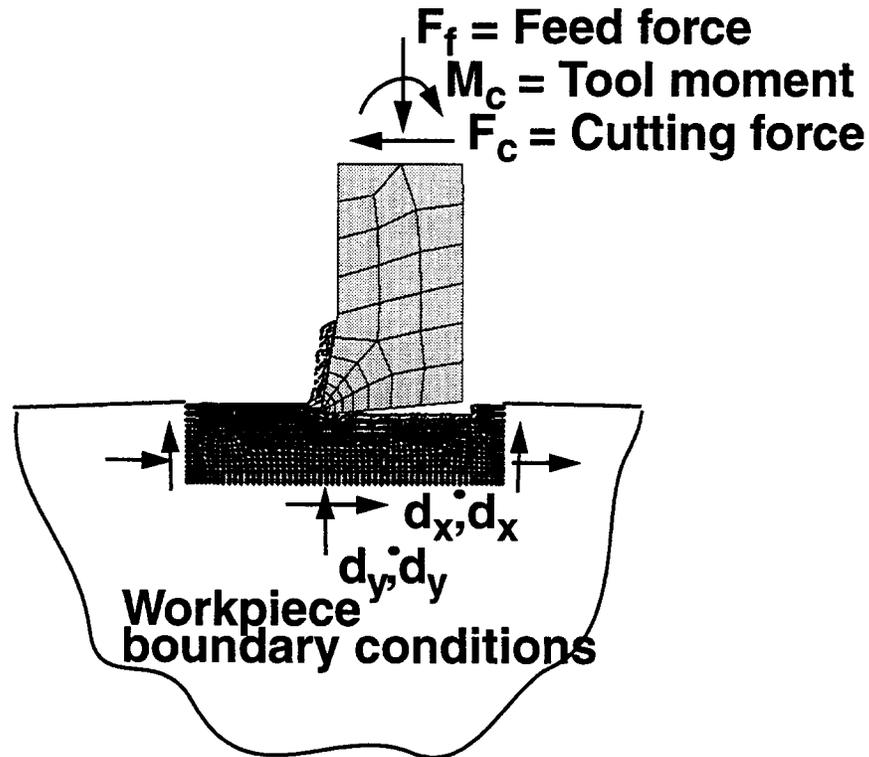


Figure 1. Smooth Particle Hydrodynamics (SPH) is used to solve the governing equations in the vicinity of the cutting process. Feed force, cutting force and cutting tool moment reactions are evaluated at boundaries that are far from the cutting interface

In the simulations done here, the tool is assumed to be elastic with properties: elastic modulus  $E=30E6$  psi, Poisson ratio  $\nu = 0.3$ . The aluminum 6061-T6 work-piece material is modeled as elastic, hardening-plastic. We note that, like the finite element method, the SPH method will admit any reasonable (or unreasonable) material constitutive model. The one used here is a power law hardening with failure model described in Stone, et. al. [11]. The criterion for material failure in this model is the equivalent plastic strain modified by the maximum tensile and the mean (hydrostatic) pressure. Importantly, this captures two well known effects: much higher likelihood of failure with positive maximum principal stress and decreased ductility in the presence of hydrostatic tension.

The cutting process simulated here includes inertial effects, but is assumed to be slow relative to thermal conduction so that temperature effects are ignored. Because of uncertainty in the nature of sliding during cutting, surface sliding is assumed to occur with a uniform

coefficient of friction  $\mu = 0.5$  in the calculations presented here.

### FE Example:

The first simulation is an illustration of the finite element method applied to the orthogonal cutting of aluminum 6061-T6 (Figure 2). This particular simulation accommodates material separation and material failure by “killing off” elements that undergo equivalent plastic strain beyond some critical level. We note that material failure characterized by the equivalent plastic strain exceeding a critical level is known to have significant error, especially under non-tensile loadings. In fact the failure criterion developed in [11] accounts for damage accumulation under general non-tensile loading conditions. However, application of this material model described in [11] to the orthogonal cutting problem resulted in severe (numerically fatal) mesh distortion when applied to this problem.

Yet even with a critical level of equivalent plastic strain dictating failure, the characteristics of element death can be observed. Figure 2 shows the sensitivity of the results to the value of that critical plastic strain. We see some amount of surface discontinuity - probably quite acceptable - due to the missing elements. Particularly in the case where elements are permitted to undergo the larger level of plastic strain, some elements suffer extreme distortion: endangering both stability and accuracy. Refinement of the mesh exacerbates this problem.

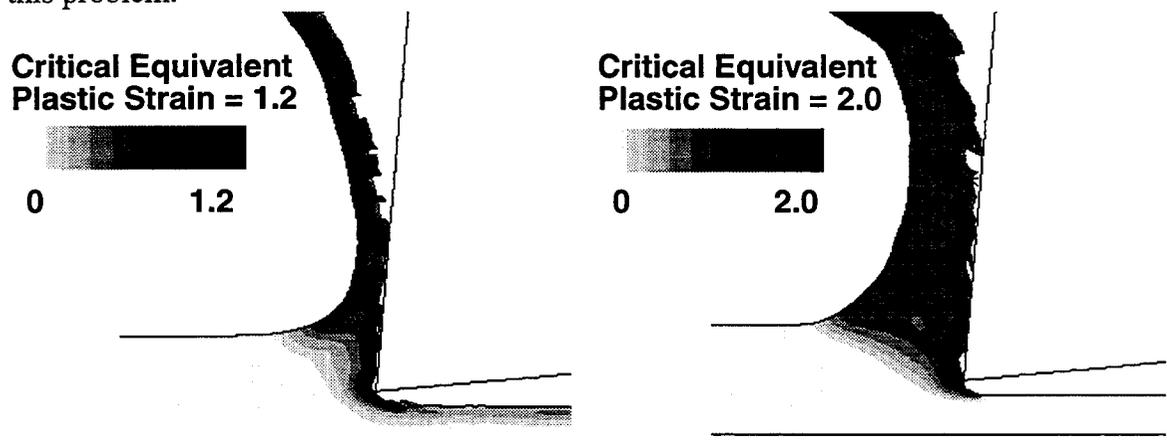


Figure 2. A finite element calculation of orthogonal cutting of Al 6061-T6 in which material separation is achieved by “killing off” elements that experience excessive plastic strain.

### SPH Example:

Figure 1, shown earlier, presents the results of using SPH to simulate the same cutting conditions. Additional cases explore the effects of rake angle and feed are shown in

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Figure 3.

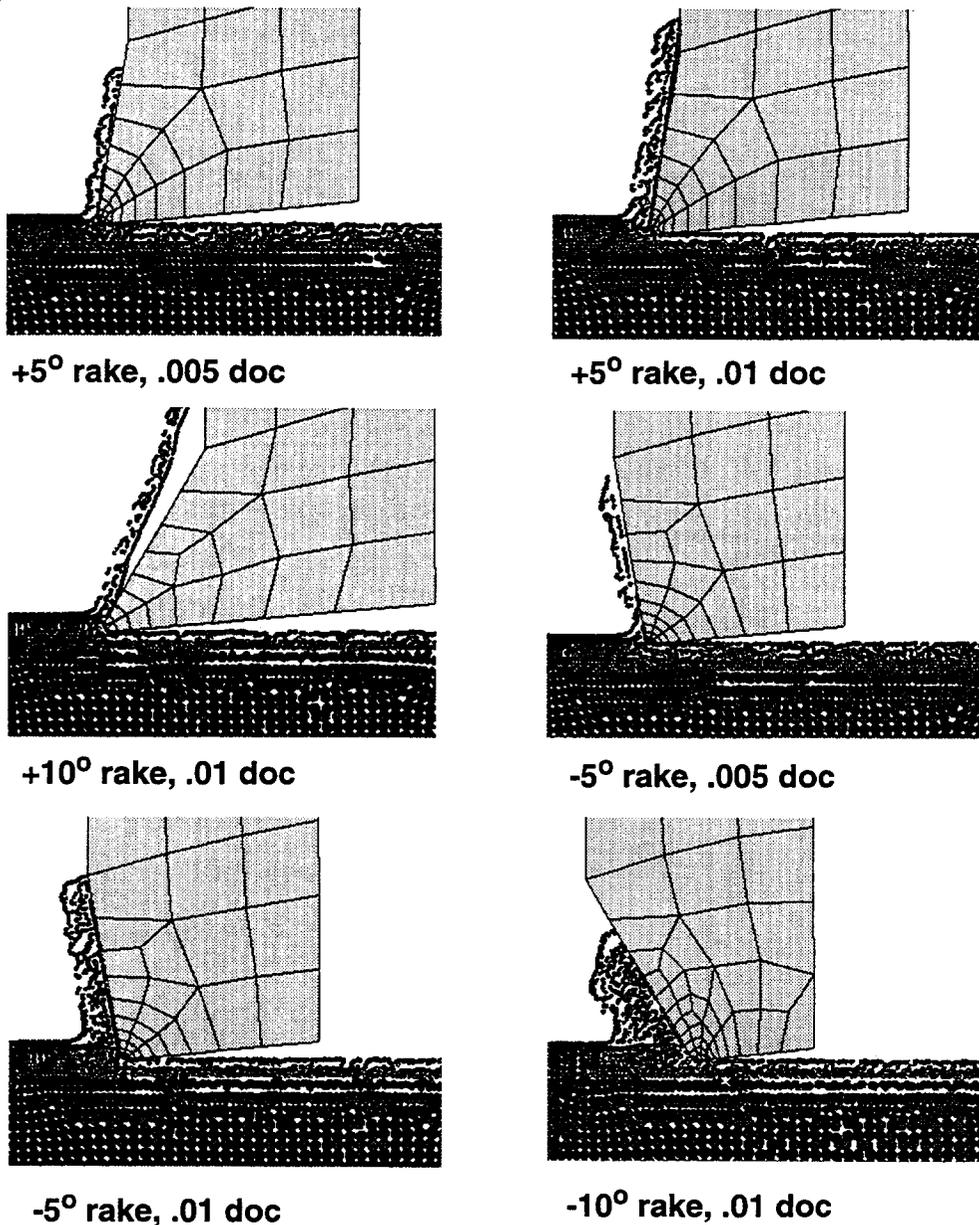


Figure 3. SPH simulation of cutting Aluminum 6061-T6 at various rake angles and feeds (sometimes referred to as depth of cut).

Note that negative rake angle is accommodated with no more difficulty than positive rake angle. In fact, Figure 4 shows a simulation with a rake angle of -45 degrees.

Figure 4 also shows some incipient cleavage lines in the substrate material about 0.025 inches below the tool tip. These physically incorrect separations are a manifestation of an instability in the SPH process discussed by Swegle et. al. [9]. Though this incipient instability would grow with time, the growth rate is slow enough that it does not interfere with the simulation. Development of methods to obviate or avoid this instability is a topic of

continuing research. The extent of equivalent plastic strain is illustrated in Figure 4 through a grey-scale spectrum on the SPH particles for the case of cutting at  $-45^\circ$ .

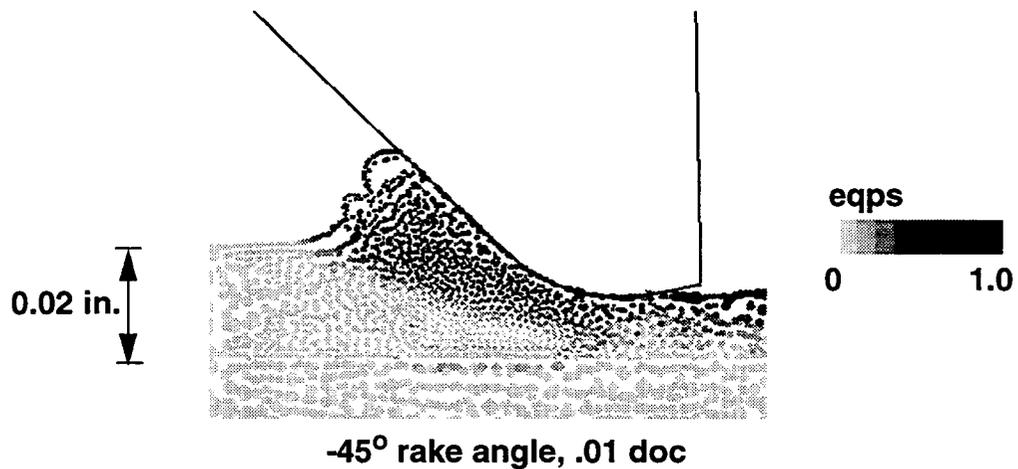


Figure 4. SPH simulation of cutting of Al 6061-T6 at a rake angle of  $-45^\circ$ .

Figures 5 and 6 present an examination of the effect of refinement of the “mesh.” In Figure 5, a positive rake angle with a rounded tip on the cutting tool is examined. Fundamentally the same kinematics are found with three different meshes. Most interestingly, a small stagnation region in front of the blunt tip is seen. It is there that the largest plastic strain and the material separation occurs.

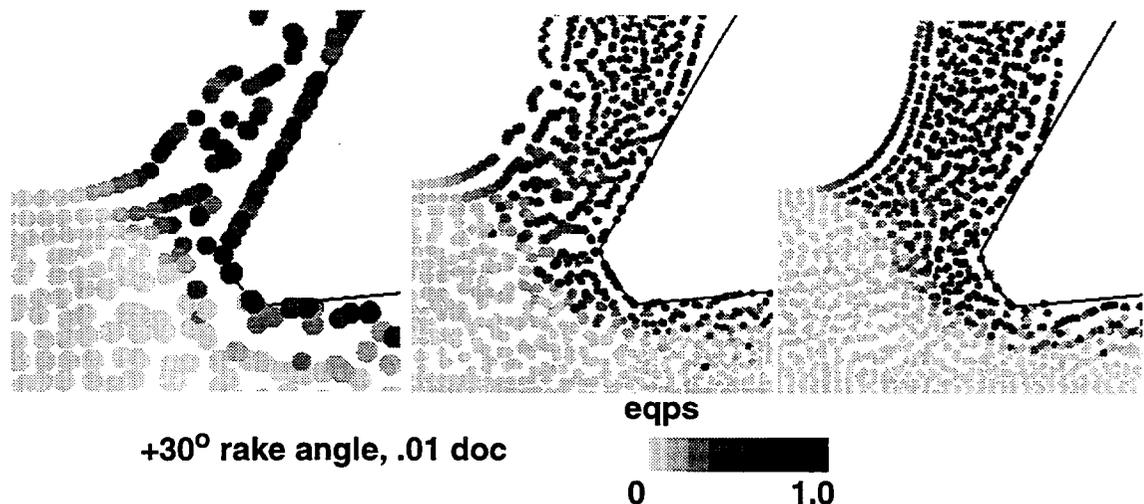


Figure 5. SPH calculations exploring the qualitative effect of mesh refinement for the case of a  $30^\circ$  rake angle.

Figure 6 examines mesh refinement for the case of a large negative rake angle. Again the kinematics are fundamentally un-affected by the mesh refinement. The region in front of the cutting tool is of special interest. There is a large band of material undergoing large

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plastic strain as the material rotates in shear in front of the cutting tool

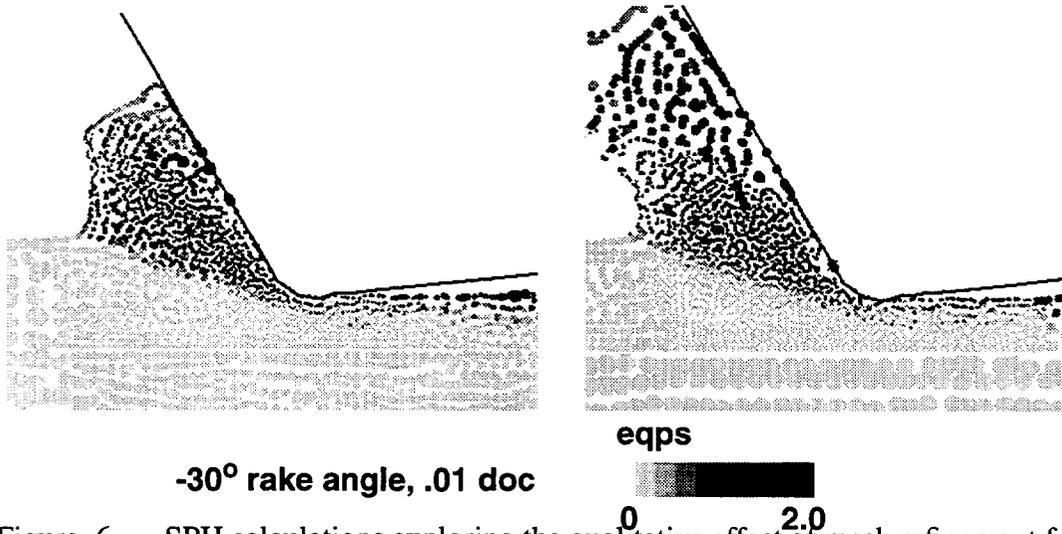


Figure 6. SPH calculations exploring the qualitative effect of mesh refinement for the case of a negative 30° rake angle.

Finally, we present the steady-state forces for the condition of +30 degree rake angle and 0.01 in. d.o.c in Figure 7 as a function of mesh refinement. One sees apparent convergence on net tool force and moment as the mesh is refined.

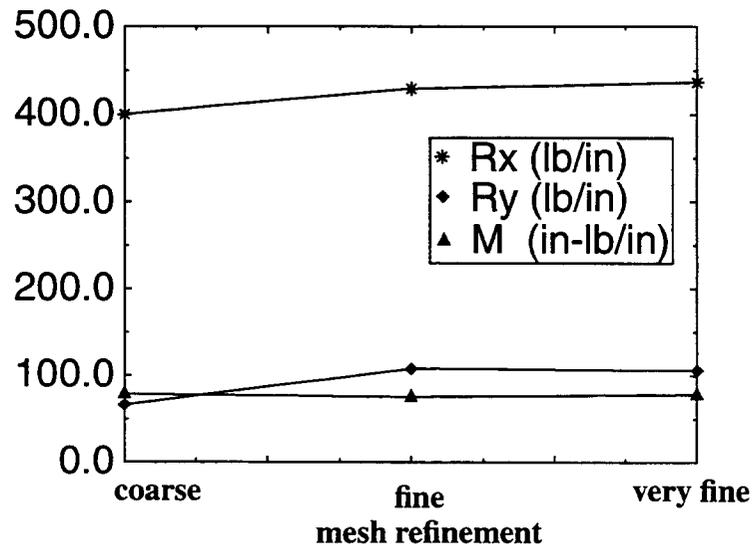


Figure 7. Convergence of the SPH calculation is investigated by examination of the convergence of reaction forces and moments on the cutting tool.

We note that qualitatively correct predictions are generated for each case of rake angle. In particular, we see normal chip formation in the cases of positive rake angle and small neg-

ative rake angle. A large negative rake angle results in the accumulation of material in front of the cutting tool. Since the actual cutting region is so small and velocities are modest, inertia should not be a significant part of the problem. With that observation, one expects the steady-state response of these calculations to be a reasonable representation of the macroscopic nature of the process.

To model the dynamics and vibration of the overall cutting process, one would use this simple nonlinear steady state model as a nonlinear interface between the dynamically linear subsystems consisting of the cutting machine, the cutting tool, and the part.

## Conclusions

The method of Smooth Particle Hydrodynamics is fairly new. Its features are not fully understood and the most effective means to exploit it are still being discovered. Despite its newness, the SPH method can be seen to be a very promising tool for the study of machining. Most importantly, this method is a tool that permits the study of the large deformations that occur near the cutting tool without the loss of accuracy and stability associated with finite element analysis of these problems.

In particular, the SPH method has been shown here to overcome the major difficulties of cutting simulation that obstruct finite element simulation of these processes. The problems of element inversion and material separation, which confound finite element analysis, are handled smoothly with the method of SPH. Another advantage demonstrated here is that mesh transition to obtain fine resolution in the vicinity of the cutting tool is achieved in a natural and easy manner.

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