Watson Scientific Computing Laboratory, N.Y.C.

Superconducting Lines for the Transmission of Large Amounts of Electrical Power over Great Distances

by

Richard L. Garwin

II. Abstract.

As a possible technological application of high-field superconductors we consider here a power line to carry 10^{11} watts a distance of 10^{3} km. (The present total installed electric power generating capacity of the U.S.A. is 2×10^{11} watts.) Such a line, in contrast to one made of ordinary metal, would dissipate essentially none of the power transmitted through it and, as proposed by J. R. Zacharias, it would allow one to burn coal near the mine and then transport the power to the larger cities and plants rather than to transport the fuel. It would allow the construction of large nuclear or solar power plants near oceans or deserts, respectively. Such a line can be made now of Nb_3Sn or Nb-Zr operating in liquid helium at 4°K. It must be used for direct current rather than alternating. The particular case discussed is for a coaxial line at 10^{5} volts and 10^{6} amperes, the cost of the line and refrigeration being ~\$3/kw (i.e. fixed charges ~0.006¢/kwh) while the saving of transport costs for coal is of the order of 0.1¢/kwh. The line thus pays for itself in ~ 3 months and saves \$1 billion/year thereafter. We have investigated in detail the problems of refrigeration along the line, as well

---

Table of Contents on Page 33.

0900615LTL

as the cooling necessitated by the heat leaking in from the wires which lead power to the consumer at room temperature. The efficiency of the line is about 99% [power transmitted less power drawn off to run refrigerator equipment, all divided by transmitted power]. While the technical discussion is probably correct, the cost figures depend strongly on the ability and the efficiency of the organization developing and constructing the line.

This is not an engineering study--it is the considered opinion of one person, and the result of some ~100 hours of direct effort on the problem. The main question is whether one wants to take the risks of interruption inherent in having a large portion of the nation's power transported on a single line, or even on an interconnected pair.

III. Introduction.

At present, the cost of the long-haul transmission of electric power (a compromise between loss of power in a small conductor and capital expense of a large diameter line) makes it uneconomical to transmit electrical power over great distances. Rather one ships the coal to the power plant at considerable expense: At the mine, coal is \$5 per ton(1) while delivered by rail it is ~\$8.50/ton.(1) (2) A modern power plant is

---

(1) "Mineral Facts and Problems," 1960(Bulletin 585, U.S. Bureau of Mines).
(2) "Electrical Transmission and Distribution Data Book," Westinghouse 1950. This book gives the cost of transport per ton as \$1.20 + 5-1/2 mills/mile. This cost then is

$$C = \$1.20 + \$3.50\ K, \qquad \text{with } K \text{ the distance in } 10^3 \text{ km.} \qquad (1)$$

~ 40% efficient in converting thermal energy of coal to electricity, i.e. ~ 1.2 kwh/lb. of coal. Thus the cost of transporting coal is \$4/ton or ~ 0.16¢/kwh, of a total production cost ~ 0.43¢/kwh in New York or Chicago.(3) This expense would be saved by generating the power at the mine or within a few miles thereof, with the coal transported by conveyer belt, at least if the cost of cooling water is not excessive.

As an example we shall discuss the question of sending half the present peak power produced by the USA a distance of 600 miles--i.e. 10^{11} watts transmitted 10^3 km or 8×10^{11} kwh/year. For a line of this capacity, the transport cost of coal required under the present system is ~ \$1 billion/year so that a considerable effort is justified to reduce this cost. One alternative, of course, is the conventional long-distance transmission of power by ordinary metallic conductors either on towers or in underground cable. The economics are discussed in Appendix G. Aside from terminal equipment, the cost of a short optimized line increases linearly with its length, as does the fraction of transmitted power dissipated in the line. Reference (2), Chapter 1, p. 5 gives the minimum cost of electrical transmission at 90% load factor as $[0.30 + 0.35 \frac{M}{100}]$ mills/kwh. Here "M" is the length of line in miles. 10^3 km thus would cost

$$C = 0.30 + 2.2 \text{ K mills/kwh} \qquad (2)$$

or ~ 0.25¢/kwh, which is why people ship coal.

---

(3) "Standard Handbook for Electrical Engineers," 9th Edition, A. E. Knowlton, Editor-in-Chief (1957), Section 10-46.

The shipment of coal does offer some advantages--(a) easy storage of energy near the consumer (no peak-load problem on the transmission system), (b) security against short-term (few day) interruptions of the transportation system. One will have to think seriously what it means to give up these virtues. Before going on to technical matters it is wise to review what every reasonable person knows: that if two things are feasible then the comparison between them must be made on the basis of cost (or return). The other fact of life is that capital is not free, and, in fact, reference (2) uses 10% annual depreciation and 6% interest as the fixed charge for equipment. Thus the constant in Eq. (2) implies a terminal facilities charge of ~ \$2.40/year/kw or an investment ~\$15/kw for terminal equipment (transformers, etc.). This fixed charge must be expected in any transmission line, superconducting or otherwise, and is less than the fixed charge required to load coal. It is just equal(4) to the cost of 2 sets of transformers (\$4.5/kw) and a set of circuit breakers (\$5/kw). The term in Eq. (2) linear in line length is a compromise between larger investment (and fixed charges) or larger losses of power. Indeed, Eq. (2) is only an approximation, the cost for long lines increasing exponentially with length beyond ~ 10^3 km. On the other hand, with superconducting lines this exponential increase occurs only beyond ~ 10^6 - 10^7 km.

---

(4) Industrial Power-Systems Handbook (Berman), 1955, p. 902.

IV. Outline of Problems Connected with a Superconducting Line.

Detailed discussion and calculations are to be found in Appendices A-I. We shall consider first a single line of great length carrying 10^6 amps at 10^5 volts. We shall describe the refrigerators required to keep it at 4°K and estimate the power required to operate the refrigerators. Next will be discussed the problem of electrically conducting, but thermally insulating, leads to the superconducting line, after which the ac-dc and dc-ac conversion machinery and the refrigerator power taps and conversion apparatus will be treated. Finally, we take up the problem of reliability, which necessitates breaking the line into many sections with frequent interconnections to a parallel line.

For the present purpose it is enough to know that certain materials ("high-field superconductors") carry current densities $> 10^5$ amp/cm^2 in transverse magnetic fields $> 10^5$ oersteds.(5) We have reason to believe that current densities exceeding 10^7 amp/cm^2 will soon be available, although superconductivity persisting to $> 10^6$ gauss is both unlikely and difficult to utilize. In what follows we plan conservatively to use $j_{\text{max.}} = 10^5$ amp/cm^2. $H_c \simeq 10^5$ gauss. Such fields produce stresses of the order of $\frac{B^2}{8\pi} \sim 400$ atm or 6000 psi, which are not excessive but must be considered. The energy density is then ~40 joules/cc and if dumped uniformly into an equal volume

---

(5) J. E. Kunzler, E. Buehler, F. S. L. Hsu, and H. Wernick, Phys. Rev. Letters 6, 89 (1961).

of superconductor would not raise its temperature to that of liquid nitrogen. The critical field and critical current density are related to the current carried on the transmission line. The power transported is, of course, still proportional to the voltage between the two conductors. Reference (3), section 13.195 describes conventional 230 kv cable with insulation 0.8" thick. We plan to use 100 kv, but the transport capacity of the line could be increased by a factor 3-5 by being slightly more venturesome. Ionic mobilities are essentially zero at 4°K, so that the insulation problem is much eased.

It is absolutely necessary (App. C) that the transmission be at dc Normal heat leak to the line is less than 3×10^{-3} watt/cm. Eddy current loss due to ac would exceed this by a factor $\sim 10^{8}$, and the energy dissipated in 120 cps vibration would also be excessive by a factor $\sim 10^{7}$. In fact, precautions must be taken against ripple fed back into the line from the inverting apparatus, in order to avoid excessive heating from the residual ac.

V. Details of proposed transmission line.

We have seen in the above that we must plan at present for conductors ~ 10 cm^2 in area to carry 10^6 amp at $< 10^5$ gauss. Eventually $\sim 10^{-1}$ cm^2 may do, but the diameter will still have to be several cm to avoid quenching by self-field. The conductors must be maintained at 4° and surrounded then by vacuum and by a 77° K N_2-cooled shield. Provision must be made for ~1/2% shrinkage of the line (see App. A), for the introduction of liquid-vapor separators every 50 m, for the operation of circulating pumps (50 watts each every 500 m) for the main refrigeration plants and power taps to run them (~3 Mw every 20 km), for vacuum pumps (every 500 m), and eventually for switching and alternate routing. If the system could be built and installed without necessity for repair, it could be made all coaxial. Experience shows otherwise, and the system shown is about as accessible as can be imagined. Vacuum joints at room temperature are made with rubber rod and 0-rings (double for leak testing). The only vacuum joints at N_2 temperature are in round copper pipe, which can be soldered or have metal flanges and gaskets. At 4°K the only critical joint is in the round copper sheath which is made in sections 50-500 m long and is joined at these intervals either by soldered sheaths of larger diameter or by flanges to permit disassembly.

To begin, let us describe a section of the line far from stations and between separators. In the Figure on p. 10 is seen the metal trough (tin-plated steel or stainless or aluminum) which is glued with epoxy resin to the concrete channel used for protection and support. The trough is tapered so that it may be stacked for shipment to the site. Alternatively, it might be rolled into the shape as it is laid. The trough sections are soldered together (or bonded with epoxy resin and the joints smoothed so as not to cause the gaskets to leak. A continuous coil of aluminum 1 mm thick x 20 km

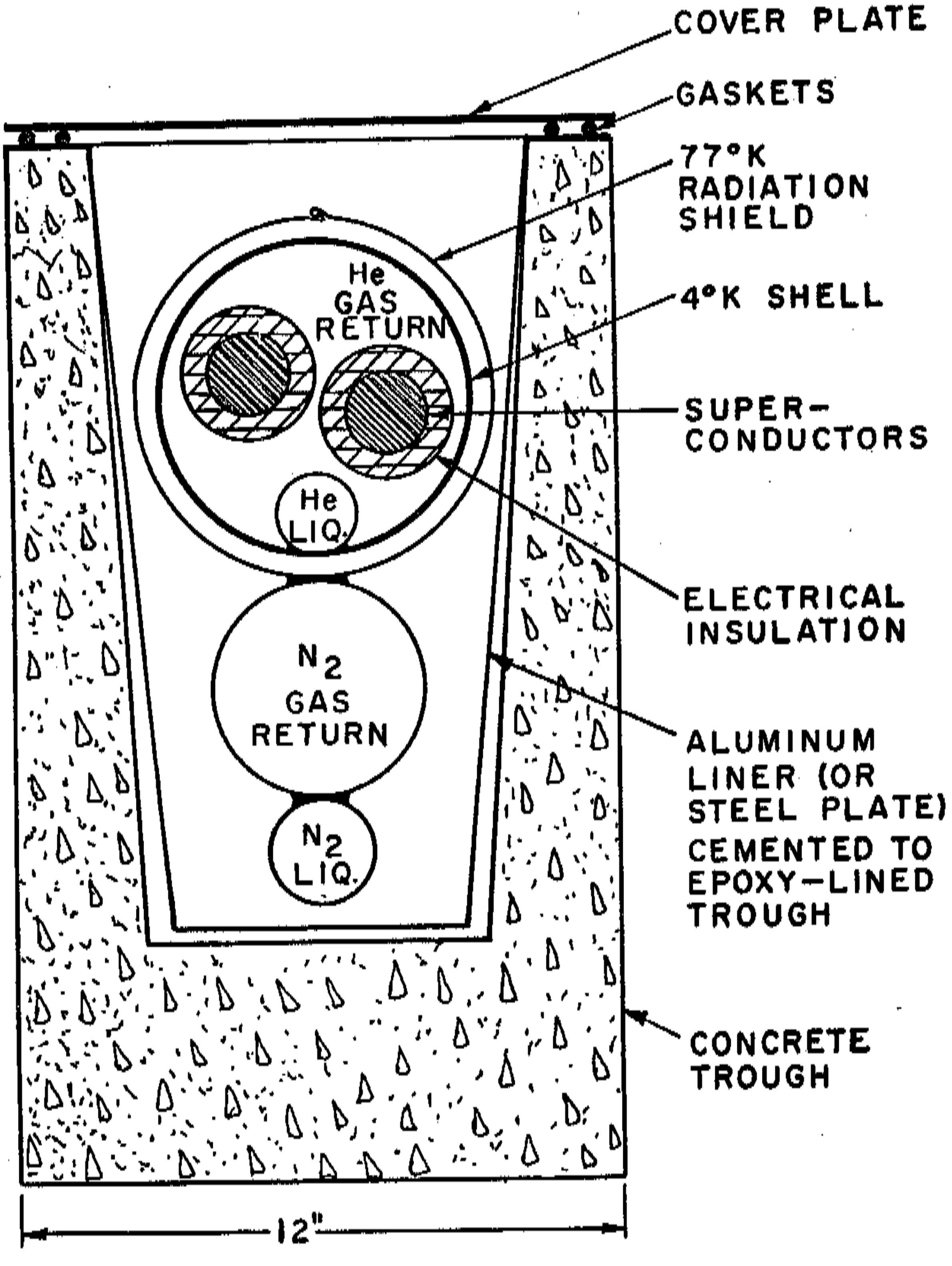

Fig. 1. Section of the superconducting line. The liquid nitrogen line and the N_2 gas return line are soldered together and to the copper-sheet radiation shield, which may be unfolded for inspection. The liquid and gas lines are connected by liquid-vapor separators at frequent intervals.

long (10 tons) is then laid on top to seal the gaskets. [This cover may be tipped up locally for inspection and repair and has occasional apertures for vacuum pumps, etc.]

Inside lie the N_2 pipe and the return manifold for vaporized N_2 gas (at 10 atm), both soldered to a sheet-metal radiation shield slit in sections ~ 10 ft. long and provided with occasional pumping holes. The N_2 shield rests on polyethylene or other cheap plastic bridges every ~3 m or so. These supports contribute negligible heat leaks. The radiation shield can be unhooked at the top seam for installation and inspection.

Surrounded by the 77°K radiation shield is the 4°K vacuum wall which contains the liquid helium line and two plastic-insulated high-field superconducting cables. These may be pulled loose into the jacket or may be supported occasionally or continuously. The free space inside this wall serves as the helium gas return. It is conventional(6) to pull 3000 ft of cable into conduit, so the projected 50 meters between separators should present no difficulty.

At intervals of ~50 meters the separator must handle an evaporation rate ~ 3 cc liquid/sec. or ~30 cc/sec. of gas under normal conditions. On cool-down the entire liquid flow-rate of the line must be allowed through the separator--clearly, it must be a device the average position of which must vary considerably but only slowly with demand. We shall not discuss this

---

(6) See Ref. 3, Section 13.198.

problem further except to note that, if necessary, the separators could include floats, switches and motors, and that they should "fail-safe" by closing.

Similarly the ~50 watt 1 ℓ/sec. circulating pumps in the lines every 500 m will not be discussed in further detail.

We are left, therefore, with the problem of describing the refrigeration stations. Cost data has been supplied by Arthur D. Little Corp. to the British "National Institute for Research in Nuclear Science" giving a price(7) ~ $\$10^6$ for a single He refrigerator to remove ~ 1 kw at 4.2°K. We note in passing that the ~500 kw motor required costs only $\$10^4$ because it is made in large numbers, and we can expect the cost of the refrigerator to be lower by a factor ~10 also if we buy 100 of them. Even at $\$10^6$ each the total for the refrigeration in the line is ~ $\$5 \times 10^7$ which is negligible.

The liquid nitrogen might be bought for the cost of the energy required to liquify it (~0.4 kwh/liter in practice). The consumption of the line would be ~100 liters/sec. or 10^7 ℓ/year. In practice, the price is about 4 times this so that the N_2 bill will be $\$40 \times 10^6$ yr.^{-1} if purchased and perhaps $\$20 \times 10^6$ if manufactured. Since we have ~ $\$10^9$ annually to work with in the savings on coal transport, we do not estimate this more accurately. Further complications to the 20 km stations come only from the desirability of switching and will be treated in detail in Appendix F.

---

(7) NIRL/R/7, Fig. 3, May, 1961.

VI. Summary.

Aside from the question of cost (see App. G), which is largely a matter of the price of high-field superconductor, which will be economical soon, we claim that it is feasible to build a line of the following characteristics:

Power transported $\equiv$ 1/2 power of USA $\equiv$ 10^{11} watt =	10^{8} kw
Voltage (dc)	10^{5} v
Current	10^{6} amp
Temperature (liquid Helium)	4.2°K
Liquid nitrogen radiation shield	77°K
Length of line	1000 km
Refrigerator spacing	20 km
Gas-liquid separator spacing	50 m
Circulating pumps (50 w each) spacing	500 m
Vacuum pump (2ℓ/sec. plus He trapping)	500 m
Thermal expansion bellows 1 m long every	500 m
[superconductors wound in helical fashion]	
Fraction of power dissipated in line and leads	$< 10^{-7}$
Fraction of power used for refrigeration	$\sim 10^{-3}$

APPENDIX A - Differential expansion.

The cryogenics engineer, like his steamy brother, must reckon with thermal expansion. Metals in being cooled to 0°K contract ~ 0.3%, some plastics ~2%. A long line fixed at the ends is thereby on cooling subjected to a 0.3% longitudinal stretch which, with an elastic modulus ~ 10^{12} dynes/cm^2 corresponds to a tension ~3000 atm or 50,000 psi. It is probably desirable to avoid such stresses and also to do away with the massive insulating clamps, etc. required to stretch the line, at the same time not communicating to it too much heat. It is not impossible to use such techniques, particularly with low thermal expansion materials, but it is desirable to have the possibility of using any reasonable materials in the line. A contraction ~ 0.3% on a 20 km run amounts to ~ 60 m, which is large. Fortunately, it is not necessary to allow for differential expansion between the 4°K and 77°K portions unless they are of very different materials. Even in that case the frequent insertion of bellows in the N_2 lines (say 0.1% bellows, or ~ 50 cm every 500 meters) will solve that problem. Undoubtedly, the simplest solution is to twist the superconducting cables as they are fed into the housing. Clearly a twist which increases the length by a few per cent will allow (preferably with a compressible center) an extension of 1/2% without much cost penalty. The large-diameter housing for the cables and for the helium line can then be equipped with bellows. Too long a run, of course, will require too much sliding on cool down, but the above seems reasonable.

APPENDIX B - Calculation of heat leak and refrigeration required.

In practice, as in theory, the heat transport will be by radiation. Per cm^2 between two surfaces of emissivity $\frac{\epsilon}{2}\,\sigma\,(T_1{}^4 - T_2{}^4)$. $\epsilon = 5\%$ is easy to achieve. A radiation shield at 77°K (liquid nitrogen temperature) is desirable (and conventional) to interrupt the radiant heat at this temperature (T_2), allowing it to be rejected at room temperature (T_1) by the expenditure of

$$W = \frac{T_1 - T_2}{T_2} \quad \frac{H}{E} \tag{b-1}$$

units of work (Carnot). In Eq. (B-1) H is the heat influx and E is the efficiency of the refrigerator (the fraction of ideal thermodynamic efficiency), which may be ~ 0.5 at 75°K, but only ~ 0.2 at ~ 4°K. It will turn out that the line is ~ 6-8 inches diameter so that the heat influx into the nitrogen shield is $\sim 1.5 \times 10^{-3}$ $w/cm^2 \times 60\ cm^2$ or $\sim 10^{-1}$ w/cm length. The radiation heat flux from 75°K to the 4° pipe is (still with $\epsilon \sim 5\%$) 6×10^{-6} $w/cm^2 \times 60\ cm^2$ or $\sim 4 \times 10^{-4}$ watts/cm length. As far as costs are concerned, a 20 km stretch of line will then have 2×10^5 watts to be rejected from 77°K and 800 watts from 4°K. The installed refrigeration capacity per 20 km will thus require 1.6 megawatts for the 77°K refrigerator and 0.25 megawatts for the 4°K refrigerator. It will be seen that ~ 1.6 Mw installed capacity for the 4° refrigerator is required to cool the 20 km section from 77°K to 4°K in one day. It will, however, require longer (say ~ 3 days) to cool to

77°K unless portable refrigerators of increased capacity are trucked in.

It is, however, not a negligible feat to cool 20 km of line from a central source. The trouble arises from the pumping power, which must also evaporate liquid. The vapor thus produced reduces the density of the mixed phase and increases the velocity necessary to maintain the same mass-flow rate. Indeed, even with the increased vapor density at low temperatures, the effect is prohibitive at both He and N_2 temperatures. We thus use separators at 50 m intervals: i.e. floats which vent trapped gas to the return manifold, while retaining liquid in the supply pipe.

Suppose we consider a pipe of radius R carrying a fluid of density ρ at temperature T. A steady heat flow "h" ergs per cm boils away liquid, which may be vented continuously into the return manifold. For simplicity take the radius independent of distance χ, and the overall length of line L.

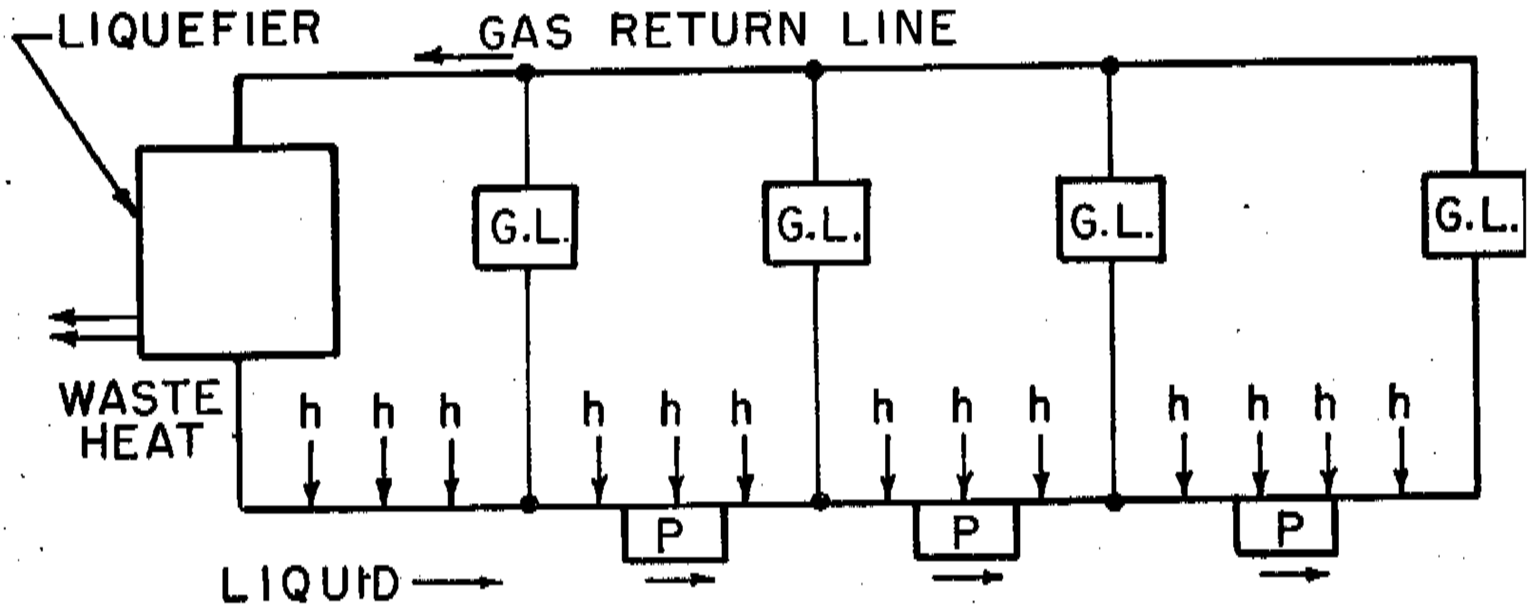

Fig. 2. Schematic of a long line receiving heat at a rate h cm^{-1} sec^{-1}, with booster circulating pumps "P" and gas-liquid separators "G. L." The separators reduce very greatly the dissipation due to increased mass velocity in the pipe, which the low-density vapor would require if allowed to accumulate in the liquid pipe. The distributed pumping "P" enables one to avoid exceeding the critical pressure of liquid helium.

We have a flow velocity "v" which varies along the pipe, being zero at the end (X = L). The volume of liquid transported is thus $V(x) = \pi R^2 v(x)$

$$\frac{dV(x)}{dx} = -\frac{1}{\lambda}\left[h + \pi R^2 v \, \frac{\rho v^2}{2} \, \frac{1}{100R}\right] \qquad \text{(b-2)}$$

λ is the heat of vaporization per cc. The first term on the right is the heat leak, while the second expresses the approximation that in turbulent flow the kinetic energy of flow is lost as heat every 50 diameters.(8)

We shall derive upper limits on the interval between refrigerators, choose a distance well within the feasible range, and determine the cool-down times. Eq. (B-2) can be written

$$-\lambda \pi R^2 \frac{dv}{dx} = h + \frac{\pi R^2 \rho v^3}{200R} \qquad \text{(b-3)}$$

or

$$-dx \, \frac{h}{\pi R^2 \lambda} = \frac{dv}{1 + \frac{\pi R \rho v^3}{200h}} \qquad \text{(b-4)}$$

With

$$b \equiv \frac{\pi R \rho}{200h} \equiv \beta^{-3} \qquad \text{(b-5)}$$

---

(8) W. H. McAdams, "Heat Transmission," McGraw-Hill (1942), pp. 118, 119.

We have for the integral of c-4

$$\frac{hL}{\pi R^2 \lambda} = 0.3\beta + \frac{\beta}{3}\left[\frac{1}{2}\ \ln\ \frac{[\beta + v(o)]^2}{\beta^2 - \beta v + v^2} + 3^{1/2}\tan^{-1}\ \frac{2v(o) - \beta}{\beta\, 3^{1/2}}\right. \qquad \text{(b-6)}$$

As v increases in Eq. (c-6), L does also, but never exceeds

$$L_{max} = \frac{\pi R^2 \lambda}{h}\ \ \frac{\pi\ 2}{3^{1/2}.\ 3} \qquad \text{(b-7)}$$

at infinite v. In fact, a practical maximum L is half that, i.e.

$$L < \frac{R^2\,\lambda\,\beta}{h} \sim \frac{R^2 \lambda}{h}\left(\frac{200h}{\pi R\ \rho}\right)^{1/3} \sim 4\lambda\, R^{5/3}\ \rho^{-1/3}\ h^{-2/3} \qquad \text{(b-8)}$$

For a line of this maximum practical length, $v \sim 0.3\beta$

Consider the liquid nitrogen problem. Here we have calculated $h \sim 10^6$ ergs sec^{-1} cm,$^{-1}$ $\rho \sim 1$, $\lambda \sim 40$ cal/cc $= 1.6 \times 10^9$ ergs/cc. Therefore, we find from Eq. (c-8) with a pipe of 2-inches diameter ($R = 2.5$ cm)

$L_{N_2} <$ 30 km

Similarly for the helium refrigeration, with $h \sim 4 \times 10^3$, $\lambda = 3 \times 10^7$ erg cm^{-3} and with a pipe of 1.5 inch diameter

$L_{He} <$ 12 km.

Thus we might well use refrigerators spaced at 20 km intervals, each one serving 10 km on either side of it. For line lengths less than this practical maximum, the required refrigeration capacity is within a factor two of the direct heat leak.

The gas-liquid separators at intervals ~ 50 m also allow one to cool down the line initially, without the usual problems of choking.(9) For example, the refrigerator and pump described above delivers liquid at a velocity $v \sim 0.3\beta$. This is a refrigeration rate $\pi R^2 v\lambda = \pi R^2 \lambda\ (0.3\beta) \sim 1$ kw at 4°K. Larger flow rates could be obtained during cool-down from the same refrigerator. This would correspond to a cooling time ~ 1 day.

Little is lost in allowing the N_2 vapor to warm to 300°K on its return to the refrigerator, since the latent heat of vaporization is equal to the energy required to cool to the boiling point. On the other hand, with helium it is quite a different story since the latent heat per cc is smaller by a factor ~60. The N_2 evaporation rate is ~ 1 liter/sec in 10 km or a gas flow rate at STP 2000 cfm. I propose to operate at 10 atm to decrease the velocity and to reduce the pumping power.

For He the critical pressure is below 2 atm. However, the vapor density at 4° is about 10% of the liquid density. A return pipe at 4°K of ten times the area will then not add much pressure drop and will only double the helium investment, which for a 1-1/2" diameter pipe 10^3 km long is about 20 million SCF, or about $\$2 \times 10^6$ and 10% of the annual production.

---

(9) "Cryogenic Engineering," R. B. Scott, Van Nostrand (1959), p. 266.

The total pressure drop in the heoium liquid line would exceed ~ 1 atm over 10 km (at the design flow velocity ~ 30 cm/sec). It is necessary to use a booster circulating pump (propeller type every 500 meters or so, to give a pressure ~ 0.05 atm at a flow rate ~ 0.1 ℓ/sec). Each of these circulating pumps consumes ~ 0.5 watts.

Thus we decide tentatively on 20 km refrigerator spacing with total installed motor power ~3 Mw at each station. Along the line we have liquid booster pumps every 500 m with liquid-vapor separators every 50 m. The He^4 gas-return line is at $4^{\circ}K$. That for N_2 may as well be at $77^{\circ}K$.

For the sake of completeness I record here that consideration has been given to cooling by supercooled N_2 and He, but these are much less effective. Superfluid He (He II) could be used without pumping, but only over much shorter runs.

Vacuum pumps: A vacuum ~ 10^{-6} mm Hg must be maintained in the insulation space. We would hope to pump out the air in ~ 1 hr., which requires perhaps ~100 second time to 1/e pressure. Since the volume of line to be pumped increases linearly with the distance between vacuum pumps, while the pumping speed is inversely as this separation, we therefore arrive at a maximum separation. The pumping spped of a pipe in cm^3/sec with D the diameter and L the length is

$$C \sim 1.2 \times 10^4 D^3 L^{-1} \tag{b-9}$$

while the volume is $V = \frac{\pi}{4} D^2 L$. Thus the pumping time constant

$$\tau \equiv \frac{V}{c} = 6 \times 10^{-5} L^2 D^{-1} \text{ sec.} \qquad \text{(b-10)}$$

For our case $D \sim 10$ so that $\tau = 100$ implies $L < 40$ m for the separation between vacuum pumps. The pumps can thus be ~ 80 m apart (we choose 50 m) and their speed must be the volume of line in 50 m per 100 sec or ~ 50 ℓ/sec. It is probably most reliable to use the liquid helium line as a trap for all gas but helium, and a low-speed ion pump for any helium which may leak in. In this fashion one does not need a mechanical pump every 50 meters. At ~ $\$5 \times 10^6$ for vacuum, we can afford ~ \$250/pumping station. Many ion pumps can be operated from the same power supply. We need only string a wire for ~2 kv dc.

At the 20 km stations it is necessary to have vacuum-tight double diaphragms to allow isolation and repair of a line section. Polyethylene or mylar are suitable and conventional.

APPENDIX C - Eddy-current losses on ac.

It would be convenient to transmit 60 cps ac for ease in tapping power, etc. This is not possible for several reasons:

(1) The characteristic impedance of the line is ~ 100 ohms, while the current and voltage limitations require a load $\sim 0.1\ \Omega$. Since the line has a length ~ 1/3 wavelength, its inductive resistance would be enormous and the required voltage $\sim 10^8$ volts rather than 10^5, and

(2) The eddy currents produced by the changing magnetic field would cause unbearable power loss and refrigeration requirements.

A wire of radius R carrying current density j will have a circumferential magnetic field

$$H(r) = \frac{2 \cdot \pi r^2 j}{r} = 2\pi r j \tag{c-1}$$

at distance r from the axis. At frequency ω and for small eddy current losses there will thus be an axial electric field

$$E_z(r) = \pi r^2 j\omega \tag{c-2}$$

which produces heat at a rate

$$h(r) = \sigma E^2(r) \tag{c-3}$$

or at a total rate

$$H = \int_0^R 2\pi r h(r)\, dr = \frac{\pi^3}{3} j^2 \omega^2 \sigma R^6 \text{ ergs cm}^{-1} \text{ sec}^{-1} = \frac{\pi}{3} I^2 \omega^2 \sigma R^2 \qquad \text{(c-4)}$$

For a material of low-temperature $\sigma \sim 10^{-3}$ emu (10^{-6} Ω-cm or about the room-temperature conductivity of copper) carrying a current $I \sim 10^5$ emu and of radius $R \sim 2$ cm we expect then a dissipation from Eq. (c-4), $H \sim 6 \times 10^{12}$ $\sim 6 \times 10^5$ watt/cm. Actually, this dissipation is less than that which would be measured. Dividing the wire into (n) filaments in the same cross-section reduces the dissipation by a factor (n) only, so that ac is completely impossible.

In fact, if the dissipation due to eddy currents is not to exceed 10% of the heat influx due to radiation, i.e. $H < 4 \times 10^{-5}$ watt/cm^2, we must have, for a solid rod an ac current (ripple) less than ten amperes. This is not such a difficult condition to realize as might be imagined, because of the very high ac impedance of the line. Indeed, a major source of ripple is the ac-dc conversion apparatus (rectifiers) at the feed (and the similar equipment at the receiving terminal). If no ripple-compensating current were fed to the line, the ripple voltage could not exceed ~ 50 kv peak so that the non-resonant current into ~ 100 Ω would be only 500 amperes. With multi-phase rectification the ripple voltage goes down like the square of the number of phases involved so that with ~50 phase rectification (~83-phase rectifiers) no ripple compensation is required. The inverters for the refrigerators (30 amps dc) should introduce small ripple currents which may be reduced to zero by proper compensation.

Turn-on.

One might at this point properly worry about the dissipation even with dc, due to fluctuating demand and therefore current. Eq. (c-4) gives in this case for a current I turned on in time τ.

$$H = \frac{\pi}{3} \quad I^2 \quad \sigma R^2 \quad t^{-2} \tag{c-5}$$

which will cause dissipation equal to 10% of the heat-leak for turn-on time t = 300 seconds. Slightly staggered turn-on times can obviously be arranged so that this is no problem. Likewise individual customers of ~ 10 amperes or ~ 1 Mw can be switched on and off every millisecond without any difficulty. A dc line therefore seems feasible.

APPENDIX D - Vibration and anelastic losses on ac.

For completeness it seems necessary only to note that ac is impossible also because of the enormous dissipation due to anelasticity. The magnetic field squeezes on the wire with pressure $\frac{B^2}{8\pi} \equiv P$. With modulus of elasticity Y there is then stored energy

$$\epsilon = \frac{P^2}{2Y} \cdot \pi R^2 \text{ ergs/cm.} \qquad \text{(d-1)}$$

In plastics and even in metals at ordinary temperature this energy is only ~ 1/2 recovered--i.e., the anelasticity "a" ~ 1/2. At low temperatures, "a" will be smaller unless there is slippage, but since $\epsilon \sim \frac{(4 \times 10^8)^2}{2 \times 10^{12}} \cdot 10 \sim 10^6$ ergs/cm and since the stress goes to zero 120 times/sec, we would have a dissipation per cm

$$H = 10^8 \text{ a} = 10 \text{ a watts/cm} \qquad \text{(d-2)}$$

Thus, we require that the anelasticity $a < 10^{-5}$, which seems unlikely. This problem is avoided, along with the dissipation due to eddy currents, by the transmission of dc.

APPENDIX E - Tap-off of power and generation of ac for the operation of the refrigeration apparatus.

We are concerned in this section with the 20 km stations which must each draw ~30 amps dc from the line and convert it into 3-phase ac for the operation of the refrigeration motors. The use of dc motors is possible but undesirable. The tap itself is the source of ~ 0.3 watt heat leak into the line for which no special provision need be made. Two wires thus emerge at room temperature and at plus and minus 50 kv. Our problem is to draw 30 amperes from them which we convert with high efficiency and low cost to 60 cps ac. Many possibilities present themselves. We shall have in any case a set of controlled rectifiers (mechanical switches, ignitrons, silicon-controlled rectifiers, vacuum tubes, or transistors) which connect the primary of the step-down transformer to the line. These rectifiers are switched in synchronism. Square-wave 60 cps thus emerges from the secondary of the transformer. Three such bridge inverters, each at 10 amps dc would be required for the generation of 3-phase power, although the job could be done more expensively with extra transformers and capacitors.

The transformers for 3 Mw cost ~ $15,000 and any of the proposed controlled rectifiers are negligible in cost compared with the transformers or especially compared with the refrigeration equipment. Switch-gear at ~ $15,000 will also be necessary.

APPENDIX F - Switching and alternate routing.

As protection against interruption of the power supply, it would be wise to have a second line nearby with frequent interconnections. Since connections are expensive, as will be seen, relatively few are desirable. On the other hand, if no interconnections are made, there may be simultaneous breakdown of both lines. I have not resolved this question: the problem is to be able to isolate a defective section of line so that it can be warmed to room temperature and repaired, which requires that it not have 100 kv across it. On theother hand, too much refrigeration capacity would be required to bring the current to room temperature at every 20 km station at all times.

My tentative solution is to have both lines permanently interconnected and to isolate only a defective section for repair. This is to be done by removing the current from the line at the generator [a temporary water load during shut-down requires the boiling away of ~ 100 tons of water per second], heating electrically a special section of the line so that a prepared joint is unsoldered, thus isolating the defective section. Such an operation might require ~ 10 sec. The power can now be reapplied and the transmission resumed for the day or so required to repair and cool down the affected section.

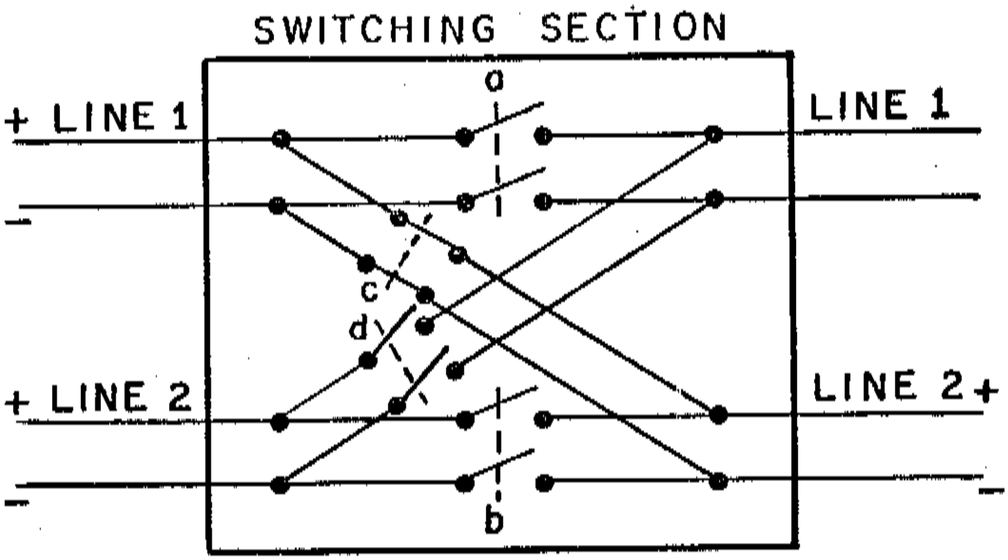

Fig. 3. A rerouting and isolating section. The region within the box is normally maintained at 4° K. When it is necessary to transfer power from line 1 on the left to line 2 on the right, the power is removed from the system, some kilowatts of heating power are coupled in to break the soldered joints in line 1 and to solder the cross-joints "c". The switching section is in a large vacuum chamber to facilitate these operations. Line 1 on the right can now be warmed to room temperature and repaired.

Such a scheme has the advantage that there is no dissipation in normal operation. It does require special sections of the line with ceramic insulation, heaters, oversized radiation shields, etc; but these problems seem soluble. Perhaps interconnections every 60 km are reasonable with every load tapped dually onto the two lines.

Joints in the line: Of course, the superconducting wire is not normally produced and shipped in infinite length. It is necessary to consider, therefore, how one makes the joints. A conductor cross-section $\sim 10\ \text{cm}^2$ means a mass/cm ~100 g, or ~ 200 tons/20 km. We must, therefore, consider joints every km for ease in shipment. A film of pure normal-metal solder (ordinary superconductors would be quenched anyhow in 10^5

gauss) 10^{-3} cm thick between butted superconducting sections $10\ \text{cm}^2$ in area, and with a resistivity at $4^\circ\text{K} \sim 10^{-8}\ \Omega$-cm, will present a resistance $\sim 10^{-12}\ \Omega$ and will cause ~ 1 watt dissipation per joint. This might be handled without excessive temperature rise ($\sim 0.5^\circ$K) but it would seem reasonable to skive the joint (cut the ends at $\sim 6^\circ$ angle to the axis of the bundle) to enable the dissipation to be reduced by a factor ~ 10, while the area of film surface for dissipation is increased by a factor 10. Alternatively, one could suffer in a skived joint $\sim 10^{-2}$ cm film thickness for 1 watt dissipation. It may be necessary to bleed liquid helium from the supply directly to those soldered joints in order to keep them at 4°K. This can be done with plastic capillaries, etc.

Power taps: McFee shows(10) that wires of optimum size extending from 4°K to 77°K dissipate and conduct 0.009 watts to the helium when one ampere is being removed and returned on a pair of leads. A 3 Mw tap for a refrigeration station means therefore a heat influx accompanying 30 amperes drain, or ~0.3 watts, which is no problem.

The end of the line is another story: Here 10^6 amps are withdrawn and the heat leak is therefore ~10 kw, which will cost us an investment ~ 5 Mw of refrigeration motor. The expense is not large, but the cooling must be good enough to prevent the heating of superconductor and the resultant spread

---

(10) McFee, R., Rev. Sci. Inst. 30, 98 (1959).

of normal phase down the line. An unstranding of the cable and a spreading of the junctions over a 10-meter length will reduce the required cooling to ~ 10 watts per cm which seems feasible on a width ~ 1 meter. There is no doubt that this can be done at reasonable cost. Such current densities have been used in superconducting magnets.

APPENDIX G - Cost of superconducting line.

Transformers and switchgear at the sending end: ascribed to the power generation cost.

	$ Million	Comments
Rectifiers--10^6 amp at 10^5 volts @ 20¢/kw	20	Upper limit
Concrete--1 yd^2 x 10^3 km @ $20/$yd^3$	20	Reasonable
Refrigeration along the line @ ~$1 M/20 km	50	"
Fabricated metal ~100 g/cm @ $1/lb	20	"
Vacuum pumps	5	"
End refrigeration	20 (?)	
Controlled rectifiers	~ 50	Unreliable
Superconductor 20 cm^2 @ ~ $50/lb	1000	Subject to rapid reduction
	$185 x 10^6	

Except for the right-of-way not discussed above, the things I have forgotten, and the cost of superconductor, the price looks reasonable. We are talking about ~10^4 tons of Nb-Zr which now sells for ~$400/lb. There is no reason why its price in large quantity should not be the same as that of its components. Similarly, we have hopes and schemes to increase the critical current density by at least a factor 10-100 which will effect a similar reduction in cost.

The above costs may be in error by large factors. Clearly the feasibility of such an enterprise depends largely on the cost and critical-current density of high-field superconductors.

APPENDIX H - Comparison with conventional super-high voltage transmission.

Conventional super-high -voltage transmission is presently being attempted at 750 kv. There is difficulty with radio interference from corona, and the right-of-way is expensive since the towers are tall. To carry 10^{11} watts 10^3 km at ~ 5% dissipation will require a current ~ 140 k amp. This would presently also be done at dc and would require two aluminum conductors ~ 2400 cm^2 each in area, for a total ~1.2 x 10^6 tons of aluminum. This is a cost ~ \$800 x 10^6 for conductors and probably several times this amount for poles ["wires" weigh ~40 tons/100 ft.]. Of course, the above line is still exacting a 5% toll on the electricity transported (say 8 x 10" kwh/yr at 0.4¢/kwh, or a total value of ~\$3 x 10^9/year).

In fact Ref. 2 (p. 5) gives the cost of the 287 kv (ac), 160 Mw, 263 mile Hoover Dam--Los Angeles line as ~ \$90/kw, while we have estimated here ~ \$8/kw for conductors in the aluminum line. I hope that this same uncertainty does not apply to my estimates of the cost of the superconducting line.

APPENDIX I - Hazards.

The energy in the line's magnetic field

$$W = 1/2\ Li^2 \equiv \int \frac{B^2}{8\pi}\ dV \qquad \text{(i-1)}$$

is that stored in 10^5 gauss and $\sim 2 \times 10^9$ cm^3, i.e. $\sim 10^{18}$ ergs. That is approximately the energy in 20 tons of high explosive, or in ~2 tons of gasoline. In case of a ground fault one can do very much better than to isolate the line and to let the energy dissipate in the fault, thus blowing up gently a few feet of line.

Peculiar effects arise with faults since the design load for the line R_o (0.1 ohm) is so low compared with the characteristic impedance Z_o (~ 100 ohm). In particular, even a dead short across the line will receive only $\frac{2V}{Z_o}$ or ~ 2000 amperes until the reflected waves are received from the terminals. If one shorts the line at the same point, the current will rise in steps of 2,000 amperes at a mean rate ~ 200,000 amp/second. The resultant arc can hardly be called catastrophic. Indeed, even if the line conductors themselves are open-circuited, the flashover will short the line and there will be plenty of time for a switch at each refrigerator station to throw ~ 50 ohms across the line. This will drain off the energy in ~ 0.1 sec., $\sim 2 \times 10^9$ joules into each resistance (~ 500° C. in 10 tons of cast-iron resistor). Further research is required into this question, but it looks very hopeful.

I. Table of Contents